

$\frac{2}{1+i}$  is a root to the quadratic equation  $z^2 + pz + q = 0$

a) Find the other root

b) Hence find the values of  $p$  and  $q$ .

$$\frac{2}{1+i}$$

Make the denominator real

$$\frac{2(1-i)}{(1+i)(1-i)} = \frac{2-2i}{2} = 1-i$$

If  $z = 1 - i$  is a root to the polynomial

Then  $z = 1 + i$  is another root

$(z - 1 + i)$  is a factor

$(z - 1 - i)$  is a factor

$$\begin{aligned}(z - 1 + i)(z - 1 - i) &= z(z - 1 - i) \\ &\quad -1(z - 1 - i) \\ &\quad +i(z - 1 - i) \\ &= z^2 - z - zi \\ &\quad -z + 1 + i \\ &\quad +zi - i - i^2 \\ &= z^2 - 2z + 2\end{aligned}$$

Hence  $p = 2$ ,  $q = 2$