

The quartic equation  $z^4 + az^3 + bz^2 + cz + d$  has roots  $2 + i$  and  $2i$

a) Work out the other roots of the equation

b) Find the values of  $a$ ,  $b$ ,  $c$  and  $d$

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a)

If  $z = 2 + i$  is a root to the polynomial

Then  $z = 2 - i$  is another root

If  $z = 2i$  is a root of the polynomial

Then  $z = -2i$  is another root

b)

$(z - 2 - i)$  is a factor

$(z - 2 + i)$  is a factor

$(z - 2i)$  is a factor

$(z + 2i)$  is a factor

$$(z - 2i)(z + 2i) = z^2 + 4$$

$$\begin{aligned}(z - 2 - i)(z - 2 + i) &= z(z - 2 + i) \\ &\quad - 2(z - 2 + i) \\ &\quad - i(z - 2 + i)\end{aligned}$$

$$= z^2 - 2z + zi$$

$$- 2z + 4 - 2i$$

$$- zi + 2i - i^2$$

$$= z^2 - 4z + 5$$

$$\begin{aligned}(z^2 + 4)(z^2 - 4z + 5) &= z^2(z^2 - 4z + 5) \\ &\quad + 4(z^2 - 4z + 5)\end{aligned}$$

$$= z^4 - 4z^3 + 5z^2$$

$$+ 4z^2 - 16z + 20$$

$$= z^4 - 4z^3 + 9z^2 - 16z + 20$$

$$\mathbf{a = -4, b = 9, c = -16, d = 20}$$