

Seven students are placed at random in a line.

- How many different arrangements are there?
- What is the probability that the two youngest students are separated?

a. Find the number of arrangements of 7 objects

$$= 7!$$

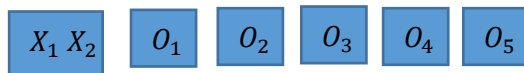
$$= 5040$$

b. **Prob**(two youngest students separated)
 $= 1 - \mathbf{Prob}$ (two youngest students are together)

Let X_1 and X_2 be the youngest students

Let O_1, O_2, O_3, O_4, O_5 be the other students

Group X_1 and X_2 together as one object.



There are effectively 6 objects here.

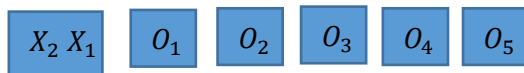
$$= 6!$$

$$= 720$$

The number of arrangement of 6 objects

BUT...

We could have X_1 and X_2 in a different order



There are 6! arrangements of this also

The total number of arrangements with two youngest students together $= 2 \times 6!$
 $= 1440$

$$\mathbf{Probability}(\text{two youngest students separated}) = \frac{1440}{5040}$$

$$= \frac{2}{7}$$

$$\mathbf{Probability}(\text{two youngest students are together}) = 1 - \frac{2}{7}$$

$$= \frac{5}{7}$$