

a) Show that $x^2 - 4x + 5$ is positive when $x = -1$

b) Prove that $x^2 - 4x + 5$ is positive for all x

a) When $x = -1$,

$$\begin{aligned}x^2 - 4x + 5 &= (-1)^2 - 4(-1) + 5 \\ &= 1 + 4 + 5 \\ &= 10\end{aligned}$$

b) To show that a quadratic expression is always positive, we can write it in the completed square form

$$x^2 - 4x + 5 \equiv (x - 2)^2 + 1$$

A square number is always positive

Therefore, $(x - 2)^2$ is always positive

...and, $(x - 2)^2 + 1$ must always be positive

Therefore, $x^2 - 4x + 5$ is always positive