Prove that the difference between the square of any two consecutive odd integers is divisible by 8

An integer is odd if it can be expressed in the form $2 n+1$

$$
\text { Let the first odd integer }=2 n+1
$$

$$
\begin{aligned}
\text { Square the first odd integer } & =(2 n+1)^{2} \\
\qquad \text { Expand and simplify } & =4 n^{2}+4 n+1
\end{aligned}
$$

The next consecutive odd integer $=2 n+3$

$$
\begin{aligned}
\text { Square the first odd integer } & =(2 n+3)^{2} \\
\text { Expand and simplify } & =4 n^{2}+12 n+9
\end{aligned}
$$

$$
\begin{aligned}
\text { Find the difference between the square numbers } & =\left(4 n^{2}+12 n+9\right)-\left(4 n^{2}+4 n+1\right) \\
& =8 n+8 \\
& =8(n+1)
\end{aligned}
$$

...which is divisible by 8

