Deductive Proofs

In proof by deduction, you start at one point, make a series of steps, where each one follows directly from the previous one and the aim is to get to end point. Often a statement will give us a starting point for the proof.

Tips

- Start with one "side" and work towards the other "side"
- Do not start with what you are trying to prove
- Keep an eye on the thing that you are trying to prove
- Take risks – if one method doesn't work, try a different one
- Practise!

Useful points to remember for deductive proofs

$\equiv$ means identically equal to

We can express an even number as $2n$

We can express an odd number as $2n - 1$

If n is a number, then the next consecutive integer is $n + 1$

If $2n$ is an even number, the next consecutive even number is $2n + 2$

If $2n - 1$ is an odd number, the next consecutive odd number is $2n + 1$

A square number can be expressed as n^2

If you can take out a common factor, then that expression must be a multiple of that factor e.g. $3x^2 + 6x$ must be a multiple of 3, since $3x^2 + 6x \equiv 3(x^2 + 2)$

If n is not prime, then $n = a \cdot b$, where $a, b \in \mathbb{Z}, 1 < a, b < n$ (in words, this means that n must have (at least) two factors that are not 1 or n)

Make sure you know the sets of numbers

$\mathbb{N}$ =natural numbers {0, 1, 2, …}
$\mathbb{Z}$ =integers {…-2, -1, 0, 1, 2, …}
$\mathbb{Q}$ =rational numbers can be expressed as $\frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$
$\overline{\mathbb{Q}}$ =irrational numbers (cannot be expressed as a rational number)
$\mathbb{R}$ =real numbers (contains all the sets above!)