The first three terms of a geometric sequence are $\log_3 x$ , $\log_9 x$ , $\log_{81} x$
Find the value of $x$ if the sum to infinity is 8.

$$\log_3 x \ , \ \log_9 x \ , \ \log_{81} x$$

$$r = \frac{U_2}{U_1}$$

$$r = \frac{\log_9 x}{\log_3 x}$$

Changing to base 3 is useful:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$r = \frac{\frac{\log_3 x}{\log_3 9}}{\log_3 x}$$

$$r = \frac{\frac{\log_3 x}{2}}{\log_3 x}$$

$$r = \frac{1}{2}$$

Sum to infinity = 8

$$S_\infty = \frac{U_1}{1 - r}$$

$$8 = \frac{U_1}{1 - \frac{1}{2}}$$

$$8 = \frac{U_1}{\frac{1}{2}}$$

$$4 = U_1$$

First term of sequence is $\log_3 x$

$$U_1 = \log_3 x$$

$$\log_3 x = 4$$

$$x = 3^4$$

$$x = 81$$

Check with sequence

$$\log_3 x \ , \ \log_9 x \ , \ \log_{81} x \ , \dots$$

$$\log_3 81 \ , \ \log_9 81 \ , \ \log_{81} 81 \ , \dots$$

$$4 \ , 2 \ , 1 \ , \dots$$