

## Proof by Contradiction

In the course, it is important to know how to do some standard proofs.

- Irrationality of square and cube roots of primes:  $\sqrt{2}, \sqrt{3}, \sqrt{5} \dots \sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5}, \dots$
- Infinite number of prime numbers

You should learn how to do these, but don't just limit yourself to them. You can learn them by heart, but it is better to understand the techniques so that you can apply them in other situations. Like many areas of mathematics, the more practice you get the better!

### The Method

1. Start with the given statement
2. Assume that the opposite of true
3. Proceed as with deductive proofs...
4. ...until this leads to a contradiction
5. Since, the opposite is false, then the statement is true

### Background Knowledge

To help you with your proofs, it is important to have some tools at your disposal:

#### Properties of numbers

Odd and Even Numbers:

$$\text{Even} + \text{Even} = \text{Even} \qquad \text{Even} \times \text{Even} = \text{Even}$$

$$\text{Even} + \text{Odd} = \text{Odd} \qquad \text{Even} \times \text{Odd} = \text{Even}$$

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#### Fundamental Theorem of Arithmetic

This is essential for proof of infinite primes and irrationality of roots of prime numbers

Any integer greater than 1, is

- either a prime number
- or can be written as a **unique product of prime numbers**

...and so, the following is helpful for proofs of the irrationality of roots of prime numbers

If  $a^n$  is divisible by  $p$  (where  $p$  is prime), then  $a$  is divisible by  $p$