Prove by induction that $\sum_{r=1}^{n} r \times 2^{r-1} = (n-1)2^n + 1$, $n \epsilon \mathbb{Z}^+$

$$\sum_{r=1}^{n} r \times 2^{r-1} = 1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \cdots + n \times 2^{n-1}$$

$$\sum_{r=1}^{n} r \times 2^{r-1} = 1 + 4 + 12 + \cdots n \times 2^{n-1}$$

Prove by induction that $1 + 4 + 12 + \cdots n \times 2^{n-1} = (n-1)2^n + 1$, $n \epsilon \mathbb{Z}^+$

Let P(n) be the proposition that $1 + 4 + 12 + \cdots n \times 2^{n-1} = (n-1)2^n + 1$, $n \epsilon \mathbb{Z}^+$

Show true for $n = 1$ $LHS = 1$

$RHS = 0 \times 2^1 + 1$

$RHS = 1$

$LHS = RHS$

Hence true for $n = 1$

Assume true for $n = k$

$$1 + 4 + 12 + \cdots k \times 2^{k-1} \equiv (k-1)2^k + 1$$

Show true for $n = k + 1$

$$1 + 4 + 12 + \cdots k \times 2^{k-1} + (k+1) \times 2^k \equiv k2^{k+1} + 1$$

$$
\begin{aligned}
LHS &\equiv 1 + 4 + 12 + \cdots k \times 2^{k-1} + (k+1) \times 2^k \\
&\equiv (k-1)2^k + 1 + (k+1) \times 2^k \\
&\equiv (k-1)2^k + (k+1)2^k + 1 \\
&\equiv 2^k(k - 1 + k + 1) + 1 \\
&\equiv 2^k(2k) + 1 \\
&\equiv k2^1 2^k + 1 \\
&\equiv k2^{k+1} + 1 \\
&\equiv RHS
\end{aligned}
$$

Hence true for $n = k + 1$

Concluding statement

True for $n = 1$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \epsilon \mathbb{Z}^+$