Prove that $sinx + sin3x + sin5x + \cdots + \sin(2n-1)\,x \equiv \frac{1-cos2nx}{2sinx} \qquad , sinx \neq 0$

Let P(n) be the proposition that $sinx + sin3x + sin5x + \cdots + \sin(2n-1)\,x \equiv \frac{1\ - cos2nx}{2sinx} \qquad , n\epsilon\mathbb{Z}^{+}\ sinx \neq 0$

Show true for $n\ = 1$ $LHS \equiv sinx$

$RHS \equiv \frac{1-cos2x}{2sinx}$

$cos2x \equiv 1 - 2sin^2x$

$1 - cos2x \equiv 2sin^2x$

$RHS \equiv \frac{2sin^2x}{2sinx}$

$RHS \equiv sinx$

$LHS \equiv RHS$

Hence true for $n\ = 1$

Assume true for $n\ = k$

$$sinx + sin3x + sin5x + \cdots + \sin(2k-1)\,x \equiv \frac{1\ - cos2kx}{2sinx}$$

Show true for $n\ =\ k+1$

$$sinx + sin3x + sin5x + \cdots + \sin(2k-1)\,x +\ \sin(2k+1)\,x \equiv \frac{1\ - cos2(k+1)x}{2sinx}$$

$$LHS\ \equiv sinx + sin3x + sin5x + \cdots + \sin(2k-1)\,x +\ \sin(2k+1)\,x$$

$$\equiv \frac{1\ - cos2kx}{2sinx}\ +\ \sin(2k+1)\,x$$

$$\equiv \frac{1\ - cos2kx}{2sinx} + \frac{\sin(2k+1)\,x \times 2sinx}{2sinx}$$

$$\equiv \frac{1\ - cos2kx + 2\sin(2kx + x)\,sinx}{2sinx}$$

$\sin(A+B) \equiv sinAcosB + cosAsinB$

$\sin(2kx + x) \equiv sin2kxcosx + cos2kxsinx$

$$\equiv \frac{1\ - cos2kx + 2(sin2kxcosx + cos2kxsinx)sinx}{2sinx}$$

$$\equiv \frac{1\ - cos2kx + 2sin2kxcosxsinx + 2cos2kxsin^2x}{2sinx}$$

$$\equiv \frac{1\ - cos2kx + sin2kx \times 2sinxcosx + cos2kx \times 2sin^2x}{2sinx}$$

$cos2x \equiv 1 - 2sin^2x$

$2sin^2x \equiv 1 - cos2x$

$sin2x \equiv 2sinxcosx$

$$\equiv \frac{1\ - cos2kx + sin2kx \times sin2x + cos2kx \times (1 - cos2x)}{2sinx}$$

$$\equiv \frac{1\ - cos2kx + sin2kx \times sin2x + cos2kx - cos2kx \times cos2x}{2sinx}$$

$$\equiv \frac{1 + sin2kx \times sin2x - cos2kx \times cos2x}{2sinx}$$

$$\equiv \frac{1 - (cos2kx \times cos2x - sin2kx \times sin2x)}{2sinx}$$

$\cos(A+B) \equiv cosAcosB - sinAsinB$

$\cos(2kx + 2x) \equiv cos2kx \times cos2x - sin2kx \times sin2x$

$$\equiv \frac{1 - (\cos(2kx + 2x))}{2sinx}$$

$$\equiv \frac{1 - \cos 2(k+1)x}{2sinx}$$

$$\equiv RHS$$

Hence true for $n = k + 1$

Concluding statement

True for $n = 1$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \epsilon \mathbb{Z}^+$