Let $z = r(cos\theta + isin\theta)$
Prove by induction that $z^n \equiv r^n[\cos(n\theta) + isin(n\theta)]$, $n\epsilon\mathbb{Z}^+$

Let P(n) be the proposition that $z^n \equiv r^n[\cos(n\theta) + isin(n\theta)], n\epsilon\mathbb{Z}^+$

Show true for $n = 2$

$$z^2 = [r(cos\theta + isin\theta)]^2$$
$$= r^2(cos\theta + isin\theta)^2$$
$$= r^2(cos^2\theta + 2cos\theta isin\theta + i^2sin^2\theta)$$
$$= r^2(cos^2\theta - sin^2\theta + i2cos\theta sin\theta)$$

$cos2\theta \equiv cos^2\theta - sin^2\theta$
$sin2\theta \equiv 2sin\theta cos\theta$

$$= r^2(cos2\theta + isin2\theta)$$

Hence true for $n = 2$

Assume true for $n = k$

$$z^k \equiv r^k[\cos(k\theta) + isin(k\theta)]$$

Show true for $n = k + 1$

$$z^{k+1} \equiv r^{k+1}[\cos((k+1)\theta) + isin((k+1)\theta)]$$

$$LHS \equiv z^{k+1}$$
$$\equiv z^1 \cdot z^k$$
$$\equiv z^1 r^k[\cos(k\theta) + isin(k\theta)]$$
$$\equiv r(cos\theta + isin\theta)r^k[\cos(k\theta) + isin(k\theta)]$$
$$\equiv r \cdot r^k(cos\theta + isin\theta)[\cos(k\theta) + isin(k\theta)]$$
$$\equiv r^{k+1}[cos\theta\cos(k\theta) + icos\theta sin(k\theta) + isin\theta\cos(k\theta) + i^2sin\theta sin(k\theta)]$$
$$\equiv r^{k+1}[cos\theta\cos(k\theta) - sin\theta sin(k\theta) + i(cos\theta sin(k\theta) + sin\theta\cos(k\theta))]$$

$\cos(A+B) \equiv cosAcosB - sinAsinB$
$\sin(A+B) \equiv sinAcosB - cosAsinB$

$$\equiv r^{k+1}[\cos(\theta + k\theta) + i\sin(\theta + k\theta)]$$
$$\equiv r^{k+1}[\cos((k+1)\theta) + isin((k+1)\theta)]$$
$$\equiv RHS$$

Hence true for $n = k + 1$

Concluding statement

True for $n = 2$
Assuming it is true for $n = k$ then it is true for $n = k + 1$
Therefore it is true for all $n \in \mathbb{Z}^+$