

Proof by Induction – Inequalities

Prove $n! > 2^n$ for $n \geq 4$



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1) Set up the proposition

Let $P(n)$ be the proposition $n! > 2^n$ for $n \geq 4$

2) Show true for starting value

$$\begin{array}{l} \text{For } n = 4 : \text{LHS} = 4! \\ \quad \quad \quad = 4 \times 3 \times 2 \times 1 \\ \quad \quad \quad = 24 \end{array} \qquad \begin{array}{l} \text{RHS} = 2^4 \\ \quad \quad = 16 \end{array}$$

3) Assume true for $n = k$

Assume $k! > 2^k$ is true for $k \geq 4$

4) Prove true for $n = k+1$

Prove $(k+1)! > 2^{k+1}$ is true for $k \geq 4$

$$\text{LHS} \equiv (k+1)k!$$

$$\text{LHS} > (k+1)2^k$$

$$\text{LHS} > (2)2^k$$

$$\text{LHS} > 2^{k+1}$$

$$\text{LHS} > \text{RHS}$$

5) Write conclusion

$P(4)$ is true

If $P(k)$ is true then $P(k+1)$ is true

Hence $P(n)$ is true for $n \geq 4$