

The sum of the first 11 terms of an arithmetic series is 3 times the sum of the first 5 terms. The 8th term is 53. Find the common difference.

$$S_n = \frac{n}{2}(2U_1 + (n - 1)d)$$

$$S_{11} = \frac{11}{2}(2U_1 + 10d)$$

$$S_{11} = 11(U_1 + 5d)$$

$$S_5 = \frac{5}{2}(2U_1 + 4d)$$

$$S_5 = 5(U_1 + 2d)$$

$$S_{11} = 3S_5$$

$$11(U_1 + 5d) = 3 \times 5(U_1 + 2d)$$

$$11(U_1 + 5d) = 15(U_1 + 2d)$$

$$11U_1 + 55d = 15U_1 + 30d$$

$$25d = 4U_1$$

$$\frac{25}{4}d = U_1$$

$$U_n = U_1 + (n - 1)d$$

$$53 = U_1 + 7d$$

$$53 = \frac{25}{4}d + 7d$$

$$112 = 25d + 28d$$

$$112 = 53d$$

$$4 = d$$