An arithmetic sequence has first term U_1 and common difference d. The sum of the first 17 terms is 136.

a) Show that $U_1 + 8d = 8$

The sum of the 2nd and the 3rd terms is 42.

b) Find d.

The nth term of the sequence is U_n.

c) Find the value of $\sum_4^{17} U_n$

a)

The sum of the first 17 terms is 136.

$$S_n = \frac{n}{2}(2U_1 + (n-1)d)$$

$$136 = \frac{17}{2}(2U_1 + (17-1)d)$$

$$136 = \frac{17}{2}(2U_1 + 16d)$$

$$136 = 17(U_1 + 8d)$$

$$\frac{136}{17} = U_1 + 8d$$

$$8 = U_1 + 8d$$

b)

The sum of the 2nd and the 3rd terms is 42.

$$U_2 + U_3 = 42$$

$$U_1 + d + U_1 + 2d = 42$$

$$2U_1 + 3d = 42$$

Solve the simultaneous equations

$$U_1 + 8d = 8$$
$$2U_1 + 3d = 42$$

Eliminate U_1

$$2U_1 + 16d = 16$$
$$2U_1 + 3d = 42$$
$$13d = -26$$
$$d = -2$$

c) Find the value of

$$\sum_4^{17} U_n$$

This is
sum of the first 17 terms - sum of the first 3 terms

Sum of first 17 terms $= 136$

$$\sum_1^{17} U_n = 136$$

$\sum_{1}^{3} U_n = U_1 + U_2 + U_3$

Sum of first 3 terms $= U_1 + U_2 + U_3$

The sum of the 2nd and the 3rd terms is 42.

Sum of first 3 terms $= U_1 + 42$

Find U_1

$8 = U_1 + 8d$

$8 = U_1 + 8(-2)$

$24 = U_1$

Sum of first 3 terms $= 24 + 42 = 66$

$\sum_{4}^{17} U_n = 136 - 66 = \mathbf{70}$