The first terms of a sequence are $\log_3 x$ , $\log_3 x^2$ , $\log_3 x^3$ , ...

Find $\boldsymbol{x}$ if the sum of the first 9 terms is 135

$log_3x \; , log_3x^2 \; , log_3x^2, \ldots$

The terms in this sequence are **arithmetic**
Consider the first two terms

$d = \; log_3x^2 - log_3x$

Use log laws to simplify:
$log_c a - log_c b = log_c \frac{a}{b}$

$d = \; log_3 \frac{x^2}{x} = log_3x$

Check with 2nd and 3rd terms

d = $log_3x^3 - log_3x^2 = log_3 \frac{x^3}{x^2} = \; log_3x$

Common difference = $log_3x$

We are told that the sum of the first 9 terms is 135
We know
$d = log_3x$
$U_1 = log_3x$

$S_n = \frac{n}{2}(2U_1 + (n-1)d)$

$135 = \frac{9}{2}(2log_3x + (9-1)log_3x)$

$135 = \frac{9}{2}(2log_3x + 8log_3x)$

$135 = \frac{9}{2}(10log_3x)$

$135 = 45log_3x$

$\frac{135}{45} = log_3x$

$3 = log_3x$

$x = 3^3$

$\boldsymbol{x = 27}$

Substitute value back into the question to check...

$log_3x \; , log_3x^2 \; , log_3x^2, \ldots$

$log_3 27 \; , log_3 27^2 \; , log_3 27^3, \ldots$

3 , 6 , 9, ...

3 + 6 + 9 + ... + 27 = 135