Three terms of a geometric sequence are $x+6\,,12\,,x-1$

Find the possible values of x

$x+6\,,12\,,x-1$

$$r=\frac{U_2}{U_1}$$

$$r=\frac{12}{x+6}$$

$$r=\frac{U_3}{U_2}$$

$$r=\frac{x-1}{12}$$

$$\frac{12}{x+6}=\frac{x-1}{12}$$

Solve the equation to find x

$$12^2=(x-1)(x+6)$$
$$144=x^2+6x-x-6$$
$$144=x^2+5x-6$$
$$0=x^2+5x-150$$

Factorise

$$0=(x+15)(x-10)$$
$$\boldsymbol{x=-15,x=10}$$

Check that the answers make sense

$x+6\,,12\,,x-1$

$x=\mathbf{-15}$
$\mathbf{-15}+6\,,12\,,\mathbf{-15}-1$
$-9,12,-16$

$$r=-\frac{4}{3}$$

$x=\mathbf{10}$
$\mathbf{10}+6\,,12\,,\mathbf{10}-1$
$16,12,9$

$$r=\frac{3}{4}$$