

$U_1 = \cos x$, $U_2 = \sin 2x$ are the first two terms of a geometric sequence, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

a) Find U_3

b) Find the set of values of x for which the geometric series converges

a) $\sin 2x \equiv 2 \sin x \cos x$

$$U_1 = \cos x$$

$$U_2 = 2 \sin x \cos x$$

$$r = \frac{U_2}{U_1}$$

$$r = \frac{2 \sin x \cos x}{\cos x}$$

$$r = 2 \sin x$$

$$U_3 = U_2 \times r$$

$$U_3 = 2 \sin x \cos x \times 2 \sin x$$

$$U_3 = 4 \sin^2 x \cos x$$

$$U_3 = 4(1 - \cos^2 x) \cos x$$

$$U_3 = 4 - 4 \cos^3 x$$

b) For a convergent series $-1 < r < 1$

$$-1 < 2 \sin x < 1$$

$$-0.5 < \sin x < 0.5$$

Consider the graph of $y = \sin x$

$$-\frac{\pi}{6} < x < \frac{\pi}{6}$$

