a) Jessica takes out a loan of $200 000 to buy an appartment. The interest rate is 4% and is calculated at the end of each year. Calculate to the nearest dollar the amount Jessica would owe the bank after 15 years.
b) In order to pay of the loan, she pays $***P*** into a bank at the end of **each** year. She receives an interest rate of 2.5% per year. Find the amount saved after 15 years.
c) What must be the value of ***P*** so that she has saved enough money to pay off the loan.

a)

After 1 year she owes
$200\,000 \times 1.04$

After 2 years she owes
$(200\,000 \times 1.04) \times 1.04$

…

After 15 years she owes
$200\,000 \times 1.04^{15}$

=$360 189

b)

After 1 year she saves
P

After 2 years she saves
$P \times 1.025 + P$

After 3 years she saves
$(P \times 1.025 + P) \times 1.025 + P$
$=1.025P^2 + 1.025P + P$

…

After 15 years she saves
$=1.025P^{14} + 1.025P^{13} + \cdots + 1.025P + P$

This is a geometric series, let's turn it around

$=P + 1.025P^2 + \cdots + 1.025P^{14}$

$U_1 = P$
$r = 1.025$
$n = 15$

Find S_{15}

$$S_n = \frac{U_n(r^n - 1)}{r - 1}$$
$$S_{15} = \frac{P(1.025^{15} - 1)}{1.025 - 1}$$
$$S_{15} = \frac{P(1.025^{15} - 1)}{0.025}$$

c)

In order to pay of the loan
Amount of savings = Amount of loan

S_{15} =\$360 189

$\frac{P(1.025^{15}-1)}{0.025}$ =360 189

$P(1.025^{15} - 1) = 360\ 189 \times 0.025$

$P = \frac{360\ 189 \times 0.025}{1.025^{15} - 1}$

$P = \$20\ 086$