A function is given by $f(x) = -x^3 + 6x^2 + 4$

a) Find the coordinates of any stationary points and describe their nature
b) Determine the values of x such that *f(x)* is a increasing function
c) Find the coordinates of the point of inflexion

$a)$

$f(x) = -x^3 + 6x^2 + 4$

Differentiate to find $f'(x)$

$f'(x) = -3x^2 + 12x$

Solve $f'(x) = 0$

$-3x^2 + 12x = 0$

$3x(-x + 4) = 0$

$x = 0, x = 4$

Find y coordinates

$f(0) = -0^3 + 6 \cdot 0^2 + 4 = 4$
$f(4) = -4^3 + 6 \cdot 4^2 + 4 = 36$

(0,4) and (4,36)

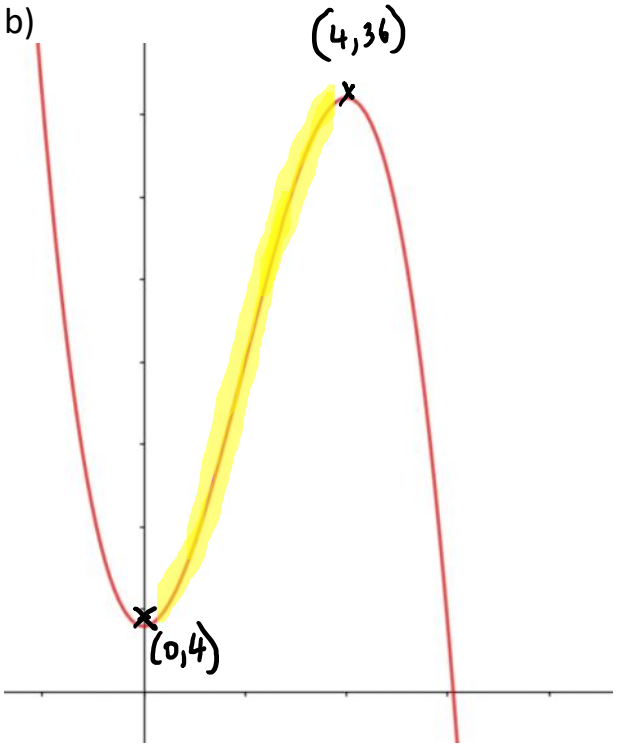
Determine their nature
Differentiate to find $f''(x)$

$f''(x) = -6x + 12$

$f''(0) = -6(0) + 12 = 12$ $\quad f''(0) > 0 \quad \Rightarrow maximum$
$f''(4) = -6(4) + 12 = -12$ $\quad f''(0) < 0 \quad \Rightarrow minimum$

Maximum at (0,4)
Minimum at (4,36)

b)



Function is increasing where $f'(x) > 0$

Function is increasing where 0<x<4

c)

Solve $f''(x) = 0$

$-6x + 12 = 0$
$-6x = -12$
$x = 2$

Since x=2 is not a stationary point, we know that it is a non-stationary point of inflexion

Find y coordinate

$f(2) = -(2)^3 + 6(2)^2 + 4 = 20$

(2, 20)