Consider the function $f(x) = -x^3 - 3x^2 + 9x$

a) Find the coordinates of any stationary points and determine their nature
b) Find the equation of the straight line that passes through both the local maximum and the local minimum points.
c) Show that the point of inflexion lies on this line.

a)

$f(x) = -x^3 - 3x^2 + 9x$

Find $f'(x)$

$f'(x) = -3x^2 - 6x + 9$

Stationary points occur where $f'(x) = 0$

$-3x^2 - 6x + 9 = 0$

$-3(x^2 + 2x - 3) = 0$

$x^2 + 2x - 3 = 0$

Factorise

and

$(x - 1)(x + 3) = 0$

$x = 1$ and $x = -3$

Find the y coordinates

$f(1) = -(1)^3 - 3(1)^2 + 9(1)$
$f(1) = 5$

$f(-3) = -(-3)^3 - 3(-3)^2 + 9(-3)$
$f(-3) = -27$

(1,5) and (-3,-27)

Find $f''(x)$

$f'(x) = -3x^2 - 6x + 9$
$f''(x) = -6x - 6$

$f''(x) < 0 \Rightarrow$ local maximum
$f''(x) > 0 \Rightarrow$ local minimum

$f''(1) = -6(1) - 6 < 0 \quad \Rightarrow$ local maximum

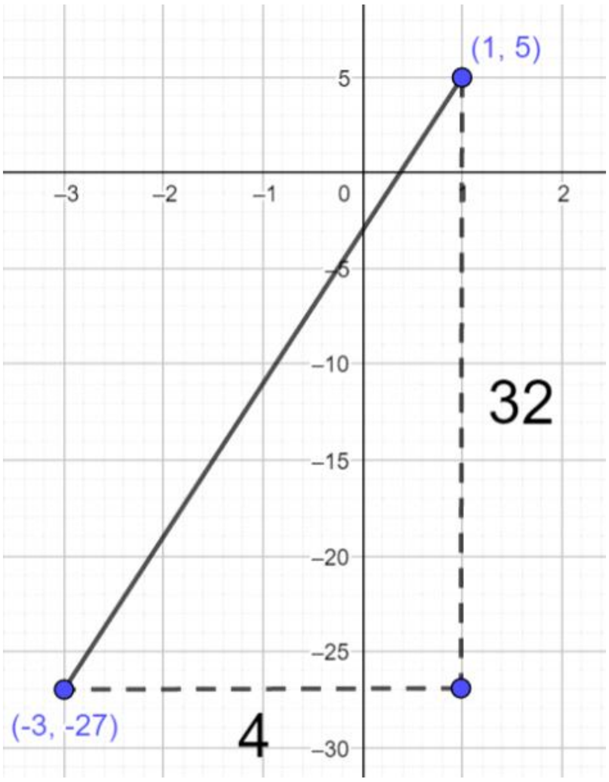
$f''(-3) = -6(-3) - 6 > 0 \quad \Rightarrow$ local minimum

local maximum at (1,5)

local minimum at (-3,-27)

b)

Find the equation of the straight line joining (1,5) and (-3,-27)



Gradient = $\frac{32}{4} = 8$

$y = mx + c$

$y = 8x + c$

Line goes through the point (1,5)

$5 = 8(1) + c$

$c = -3$

$y = 8x - 3$

c)

$f''(x) = -6x - 6$

Solve $f''(x) = 0$

$-6x - 6 = 0$

$x = 1$

Since $f'(1) \neq 0$ then x=1 is not a stationary point

Find y coordinate

$f(1) = -(1)^3 - 3(1)^2 + 9(1)$

$f(1) = 5$

Point of inflexion at (1,5)

Show that (1,5) lies on $y = 8x - 3$

$y = 8(1) - 3$

$y = 5$