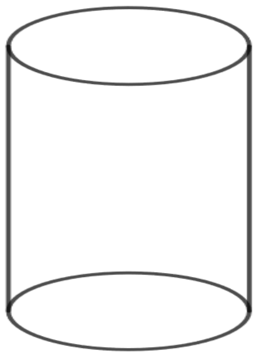
An **open** cylindrical tin is to be made from a large sheet of metal.

1. Write an expression for the volume of the tin in terms of ***r***, its radius and ***h***, its height.
2. The tin should have a volume of $\frac{\pi}{8}\ m^3$. Write an expression for the ***h*** in terms of ***r***.
3. Show that the total surface area of the tin, A can be written

$$A = \pi r^2 + \frac{\pi}{4r}$$

4. To minimize costs, the tin should be made with the minimum possible surface area.
   Find $\frac{dA}{dx}$ and hence find the dimensions of the tin which make the area a minimum.
5. Find $\frac{d^2A}{dx^2}$ and justify that these dimensions give the minimum area.

1.



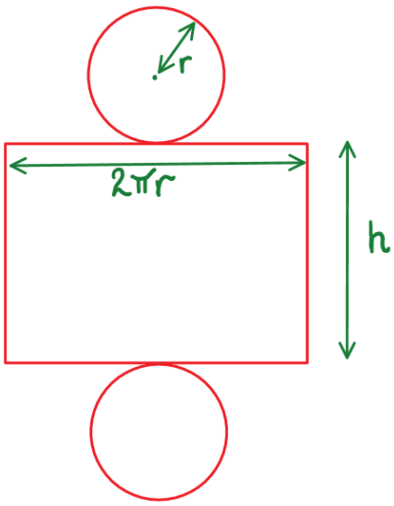
$$V = \pi r^2 h$$

2.

$$\pi r^2 h = \frac{\pi}{8}$$

$$h = \frac{1}{8r^2}$$

3.



$$A = 2\pi r^2 + 2\pi r h$$

$$A = 2\pi r^2 + 2\pi r \frac{1}{8r^2}$$

$$A = \pi r^2 + \frac{\pi}{4r}$$

4. Minimum occurs where $\frac{dA}{dr} = 0$

$$A = \pi r^2 + \frac{\pi}{4} r^{-1}$$

$$\frac{dA}{dr} = 2\pi \mathrm{r} - \frac{\pi}{4} r^{-2}$$

$$2\pi \mathrm{r} - \frac{\pi}{4\mathrm{r}^2} = 0$$

$$2\pi \mathrm{r} = \frac{\pi}{4\mathrm{r}^2}$$

$$r^3 = \frac{1}{8}$$

$$r = \sqrt[3]{\frac{1}{8}}$$

$$r = \frac{1}{2}$$

$$h = \frac{1}{8r^2}$$

$$h = \frac{1}{8\left(\frac{1}{2}\right)^2}$$

$$h = \frac{1}{8 \cdot \frac{1}{4}}$$

$$h = \frac{1}{2}$$

5.

$$\frac{dA}{dr} = 2\pi \mathrm{r} - \frac{\pi}{4} r^{-2}$$

$$\frac{d^2A}{dr^2} = 2\pi + 2\frac{\pi}{4} r^{-3}$$

$$\frac{d^2A}{dr^2} = 2\pi + \frac{\pi}{2r^3}$$

When $r = \frac{1}{2}, \frac{d^2A}{dr^2} = 2\pi + \frac{\pi}{2\left(\frac{1}{2}\right)^3}$

$\frac{d^2A}{dr^2} > 0$, hence A is a minimum