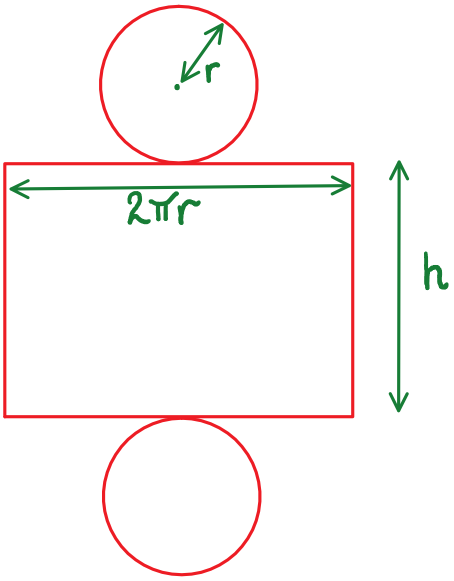
A drinks manufacturer wants a design for a new can for their mini drinks collection, volume=170ml. Your job is to find the can that will contain a volume of 170ml = 170cm^3 ≈54πcm^3 and that will minimize the amount of aluminium used.

The can is a closed cylinder of base ***r*** and height ***h*** has a volume of 54πcm^3



Surface Area $A = 2\pi r^2 + 2\pi r h$

Volume $V = \pi r^2 h$

$$54\pi = \pi r^2 h$$

$$\frac{54}{r^2} = h$$

$$A = 2\pi r^2 + 2\pi r \frac{54}{r^2}$$

$$A = 2\pi r^2 + 2\pi \frac{54}{r}$$

$$A = 2\pi r^2 + 108\pi r^{-1}$$

Minimum area occurs when $\frac{dA}{dr} = 0$

$$\frac{dA}{dr} = 4\pi r - 108\pi r^{-2}$$

$$4\pi r - \frac{108\pi}{r^2} = 0$$

$$4\pi r = \frac{108\pi}{r^2}$$

$$r^3 = \frac{108\pi}{4\pi}$$

$$r^3 = 27$$

$$r = 3$$

$$h = \frac{54}{r^2}$$

$$h = \frac{54}{3^2}$$

$$h = 6$$

Verify minimum $\frac{dA}{dr} = 4\pi r - 108\pi r^{-2}$

$$\frac{d^2A}{dr^2} = 4\pi + 216\pi r^{-3}$$

When $r = 3$ $\frac{d^2A}{dr^2} > 0\ , hence\ minimum$