Let $f(x) = x^2(2x-3)^3$

a) Find $f'(x)$

b) The graph of y = f(x) has stationary points at x = 0, x=$\frac{3}{2}$ and x = **a**. Find the value of **a**

a) $f(x) = x^2(2x-3)^3$

This is the product of 2 functions.
Before we use the product rule, we need to use the chain rule to differentiate $(2x-3)^3$

We know
$\frac{d}{dx}[f(x)]^n = nf'(x)[f(x)]^{n-1}$

$$\frac{d}{dx}[2x-3]^3 = 3(2)[2x-3]^2$$
$$= 6[2x-3]^2$$

Use the Product Rule to find $f'(x)$

$f(x) = g(x)h(x)$

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(x) = 2x(2x-3)^3 + x^2 \cdot 6(2x-3)^2$

$f'(x) = 2x(2x-3)^3 + 6x^2(2x-3)^2$

We can factorise this answer which will help up with part b

There is a common factor of $2x(2x-3)^2$

$f'(x) = 2x(2x-3)^2(2x-3+3x)$

$f'(x) = 2x(2x-3)^2(5x-3)$

b)

Stationary points are when $f'(x) = 0$

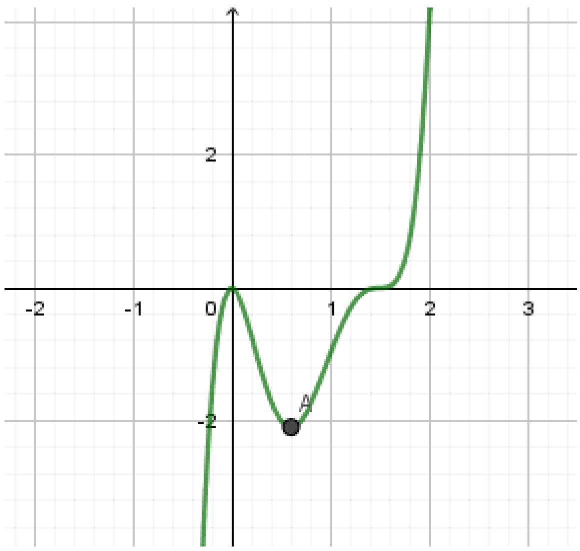
Solve $f'(x) = 0$

$2x(2x-3)^2(5x-3) = 0$

$2x = 0\ ,\ (2x-3)^2 = 0\ ,\ 5x-3 = 0$

$x = 0\ ,\ x = \frac{3}{2}\ ,\ x = \frac{3}{5}$

$y = f(x)$ has stationary points at
x = 0, x=$\frac{3}{2}$ and x = **a**



$a = \frac{3}{5}$