Let $y = xe^x$

a) Find $\frac{dy}{dx}$

b) Show that $\frac{d^2x}{dy^2} = e^x(2 + x)$

c) Find the coordinates of the stationary point and show that it is a local minimum.

a) $y = xe^x$

This is a product so we need to use the Product Rule

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + \frac{du}{dx}v$

$\frac{dy}{dx} = xe^x + 1 \cdot e^x$

$\frac{dy}{dx} = xe^x + e^x$

b) $\frac{dy}{dx} = xe^x + e^x$

Use Product Rule agin for xe^x

$\frac{d^2x}{dy^2} = xe^x + 1 \cdot e^x + e^x$

$\frac{d^2x}{dy^2} = xe^x + e^x + e^x$

$\frac{d^2x}{dy^2} = xe^x + 2e^x$

Factorise

$\frac{d^2x}{dy^2} = e^x(x + 2)$

c) $\frac{dy}{dx} = xe^x + e^x$

For stationary point solve $\frac{dy}{dx} = 0$

$xe^x + e^x = 0$

Factorise

$e^x(x + 1) = 0$

$e^x = 0$, $x + 1 = 0$

No solutions, x = -1

Justify this is a minimum

When x = -1, $\frac{d^2x}{dy^2} = e^{(-1)}(-1 + 2)$

$\frac{d^2x}{dy^2} = e^{-1}$

Since $\frac{d^2x}{dy^2} > 0$,

there is a local minimum at x = -1