Let $f(x) = tanx$. A$\left(\frac{\pi}{3}, \sqrt{3}\right)$ is a point that lies on the graph of

a) Given that $\tan x = \frac{sinx}{cosx}$ find $f'(x)$

b) Show that $f'\left(\frac{\pi}{3}\right) = 4$

c) Find the equation of the normal to the curve y = f(x) at the point A
d) Show that the normal crosses the y axis at $\sqrt{3} + \frac{\pi}{12}$

a) $\mathrm{f(x)} = \tan x = \frac{sinx}{cosx}$

This is a quotient so we need to use the Quotient Rule

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{[h(x)]^2}$$

$g(x) = sinx \quad \Rightarrow g'(x) = \cos x$

$h(x) = cosx \quad \Rightarrow h'(x) = -\sin x$

$$f'(x) = \frac{cosx\ \cos x - (-\sin x)sinx}{[cosx\ ]^2}$$

$$f'(x) = \frac{cos^2x + sin^2x}{cos^2x}$$

From the formula booklet, we know
$sin^2x + cos^2x \equiv 1$

$$f'(x) = \frac{1}{cos^2x}$$

You may write this answer as
$f'(x) = sec^2x$

b) $f'\left(\frac{\pi}{3}\right) = \frac{1}{(cos\frac{\pi}{3})^2}$

$cos\frac{\pi}{3} = cos60° = \frac{1}{2}$

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\left(\frac{1}{4}\right)} = 4$$

c) Gradient of the tangent =4

Gradient of normal = - $\frac{1}{\text{gradient of tangent}}$

Gradient of the normal = $-\frac{1}{4}$

Equation of the normal

$y = -\frac{1}{4}x + c$

$\left(\frac{\pi}{3}, \sqrt{3}\right)$

When x = $\frac{\pi}{3}$, y = $\sqrt{3}$

$\sqrt{3} = -\frac{1}{4}\left(\frac{\pi}{3}\right) + c$

$\sqrt{3} = -\frac{\pi}{12} + c$

$\sqrt{3} + \frac{\pi}{12} = c$

Equation of the normal

$y = -\frac{1}{4}x + \sqrt{3} + \frac{\pi}{12}$

d) Hence the y intercept = $\sqrt{3} + \frac{\pi}{12}$