Let $f(x) = \frac{lnx}{x}, x > 0$

a) Show that $f'(x) = \frac{1-lnx}{x^2}$

b) Find $f''(x)$

c) The graph of f has a point of inflexion at A. Find the x-coordinate of A.

a) $f(x) = \frac{lnx}{x}$

This is a quotient so we need to use the Quotient Rule

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{[h(x)]^2}$$

$$f'(x) = \frac{x \cdot \frac{1}{x} - 1 \cdot lnx}{[x]^2}$$

$$f'(x) = \frac{1 - lnx}{x^2}$$

b) $f'(x) = \frac{1 - lnx}{x^2}$

We need to use the Quotient Rule again

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1 - lnx)}{[x^2]^2}$$

$$f''(x) = \frac{-x - 2x + 2xlnx}{x^4}$$

$$f''(x) = \frac{-3x + 2xlnx}{x^4}$$

$$f''(x) = \frac{x(-3 + 2lnx)}{x^4}$$

$$f''(x) = \frac{-3 + 2lnx}{x^3}$$

c)

Solve $f''(x) = 0$

$$\frac{-3 + 2lnx}{x^3} = 0$$

Since $x \neq 0$

$$-3 + 2lnx = 0$$

$$2lnx = 3$$

$$lnx = \frac{3}{2}$$

Take inverse logs of each side of the equation

$$x = e^{\frac{3}{2}}$$