$f(x) = x^3\ (x-3)^2$

The function $\boldsymbol{f}$ has three stationary points. Find the x coordinates of these points

$f(x) = x^3\ (x-3)^2$

$f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$g(x) = x^3$ $\qquad$ $h(x) = (x-3)^2$

$\frac{d}{dx}[f(x)]^n = n \cdot f'(x) \cdot [f(x)]^{n-1}$

$g'(x) = 3x^2$ $\qquad$ $h'(x) = 2 \cdot 1(x-3)^1$

$h'(x) = 2(x-3)$

$f'(x) = g(x)h'(x) \quad + \quad g'(x)h(x)$

$f'(x) = x^3 \cdot 2(x-3) \quad + \quad 3x^2\ (x-3)^2$

$f'(x) = 2x^3(x-3) \quad + \quad 3x^2\ (x-3)^2$

Stationary points occur where $f'(x) = 0$. Factorise $f'(x)$

$f'(x) = x^2(x-3)(2x) \quad + \quad x^2(x-3)3(x-3)$

$f'(x) = x^2(x-3)(2x + 3(x-3))$

$f'(x) = x^2(x-3)(2x + 3x - 9)$

$f'(x) = x^2(x-3)(5x-9)$

Stationary points occur where $f'(x) = 0$

$x^2(x-3)(5x-9) = 0$

$x^2 = 0\,, (x-3) = 0\,, (5x-9) = 0$

$x = 0\ , \qquad x = 3\,, \qquad x = \frac{9}{5}$