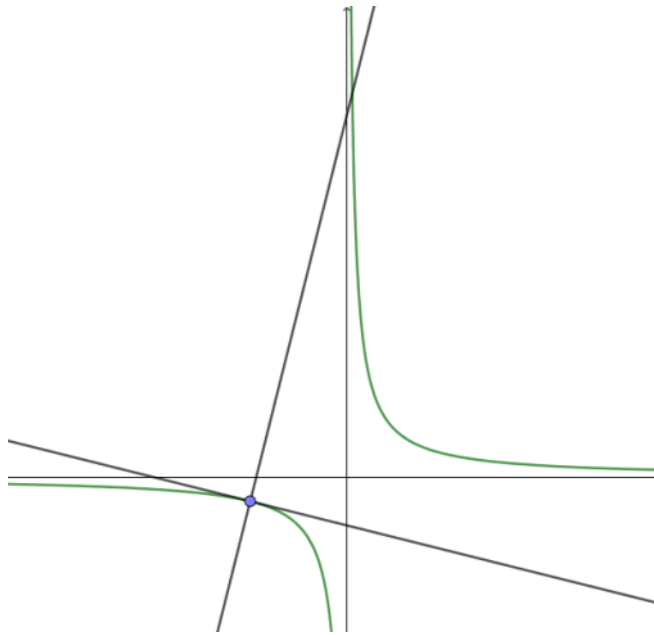a) Find the equation of the normal to the curve $f(x) = \frac{1}{x}$ at the point $P\left(-2, -\frac{1}{2}\right)$.

b) Find the co-ordinates of the point where this normal meets the curve again.

a)

$f(x) = \frac{1}{x}$

$f(x) = x^{-1}$

Find the gradient function

$f'(x) = -1x^{-2}$

$f'(x) = \frac{-1}{x^2}$

Find the gradient at the point $P\left(-2, -\frac{1}{2}\right)$

$f(\text{-2}) = \frac{-1}{(\text{-2})^2}$

$f(\text{-2}) = \frac{-1}{4}$

Tangent has gradient $= -\frac{1}{4}$

Gradient of normal $= -\dfrac{1}{\text{gradient of tangent}}$

Normal has gradient $= 4$

Equation of the normal

$y = 4x + c$

The line passes through the point $\left(-2, -\frac{1}{2}\right)$

Find c

$-\frac{1}{2} = 4(\text{-2}) + c$

$-\frac{1}{2} = -8 + c$

$$7\frac{1}{2} = c$$

$$c = \frac{15}{2}$$

$$y = 4x + \frac{15}{2}$$

$$y = \frac{1}{x}$$

$$y = 4x + \frac{15}{2}$$

Find the intersection of the curve and the normal

$$\frac{1}{x} = 4x + \frac{15}{2}$$

Multiply both sides by x

$$1 = 4x^2 + \frac{15}{2}x$$

$$2 = 8x^2 + 15x$$

$$8x^2 + 15x - 2 = 0$$

Factorise
...we know $(x + 2)$ is a factor

$$(x + 2)(8x - 1) = 0$$

Solve

$$x = -2, x = \frac{1}{8}$$

Point of intersection $\left(-\frac{1}{8}, 8\right)$