Let $f(x) = \frac{x^4-4x^2}{4}$
C(2 , 0) lies on the graph of y = f(x)

a) The tangent to the graph of y = f(x) at C cuts the y axis at A. Find the coordinates of A.
b) The normal to the graph of y = f(x) at C cuts the y axis at B. Find the area of the triangle ABC.

a)

$$f(x) = \frac{x^4 - 4x^2}{4}$$

Write as 2 separate expressions

$$f(x) = \frac{x^4}{4} - x^2$$

Differentiate to find gradient function

$$f'(x) = \frac{4x^3}{4} - 2x$$

$$f'(x) = x^3 - 2x$$

Find gradient when x=2

$$f'(2) = 2^3 - 2 \cdot 2$$

$$f'(2) = 4$$

Find the equation of the tangent
gradient = 4

$$y = 4x + c$$

Find c
Tangent passes through the point (2, 0)

$$0 = 4 \cdot 2 + c$$

$$c = -8$$

$$y = 4x - 8$$

The tangent cuts the y axis at A
x = 0

$$y = -8$$

A(0,-8)

b)

$$\text{gradient of the normal} = -\frac{1}{gradient\ of\ tangent}$$

$$m = -\frac{1}{4}$$

Find the equation of the normal

gradient = $-\frac{1}{4}$

$$y = -\frac{1}{4}x + c$$

Find c

Normal passes through the point (2, 0)

$$0 = -\frac{1}{4} \cdot 2 + c$$

$$\frac{1}{2} = c$$

$$y = -\frac{1}{4}x + \frac{1}{2}$$

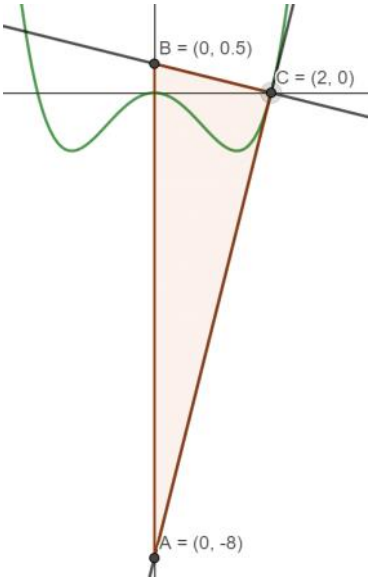
Normal cuts the y axis at B

x = 0

$$y = -\frac{1}{4} \cdot 0 + \frac{1}{2}$$

$$y = \frac{1}{2}$$

B$\left(0, \frac{1}{2}\right)$



Area = $\frac{8\frac{1}{2} \cdot 2}{2}$

Area = $8\frac{1}{2}$