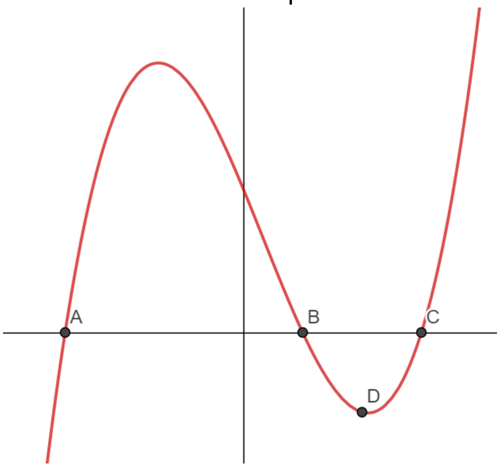
The function $f(x) = x^3 - x^2 - 9x + 9$ intersects the x axis at A, B and C.
The x coordinate of the point D is the mean of the x coordinates of B and C.



a) Find the coordinates of A, B and C.
b) Find the equation of the tangent to the curve at D.
c) Find the point of the intersection of the tangent with the curve. Interpret your result.

a)

$f(x) = x^3 - x^2 - 9x + 9$

Use Factor Theorem to find the zeros of the function f

$f(3) = (3)^3 - (3)^2 - 9(3) + 9$
$f(3) = 0$ hence $x - 3$ is a factor

$f(-3) = (-3)^3 - (-3)^2 - 9(-3) + 9$
$f(-3) = 0$ hence $x + 3$ is a factor

$(x-3)(x+3) = x^2 - 9$

Find the remainder when $x^3 - x^2 - 9x + 9$ is divided by $x^2 - 9$

$(x^2 - 9)(ax + b) \equiv x^3 - x^2 - 9x + 9$

Equate x^3 terms

$ax^3 = x^3$
$a = 1$

Equate units

$-9b = 9$
$b = 1$

Hence x + 1 is a factor

$f(x) = (x-1)(x-3)(x+3)$

A, B and C are the x intercepts

A(-3,0) , B(1,0) , C(3,0)

b)

B(1,0) , C(3,0)

Find coordinates of D

x coordinate = $\frac{1+3}{2} = 2$

y coordinate = f(2)

$f(2) = (2-1)(2-3)(2+3)$

$f(2) = (1)(-1)(5) = -5$

D(2,-5)

$f(x) = x^3 - x^2 - 9x + 9$

Find gradient function

$f'(x) = 3x^2 - 2x - 9$

Find gradient of tangent at D

$f'(2) = 3(2)^2 - 2(2) - 9$

$f'(2) = 12 - 4 - 9 = -1$

Find the equation of the tangent

gradient = -1

$y = -x + c$

Tangent passes through the point D(2,-5)

$-5 = -2 + c$

$c = -3$

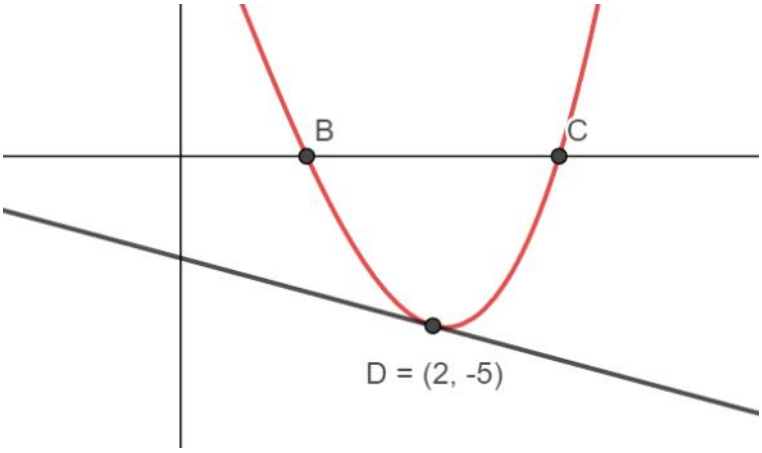
$y = -x - 3$

c)

Find intersection of $y = -x - 3$ with $y = x^3 - x^2 - 9x + 9$

$x^3 - x^2 - 9x + 9 = -x - 3$

$x^3 - x^2 - 8x + 12 = 0$



We know that $(x-2)^2$ is a factor

$(x-2)^2 = x^2 - 4x + 4$

$x^3 - x^2 - 8x + 12 = (cx + d)(x^2 - 4x + 4)$

Equate x^3 terms

$x^3 = cx^3$

$c = 1$

Equate units

$12 = 4d$

$d = 3$

$x + 3$ is the other factor

Tangent intersects with curve at $x = -3$

This is the point C

(-3 , 0)