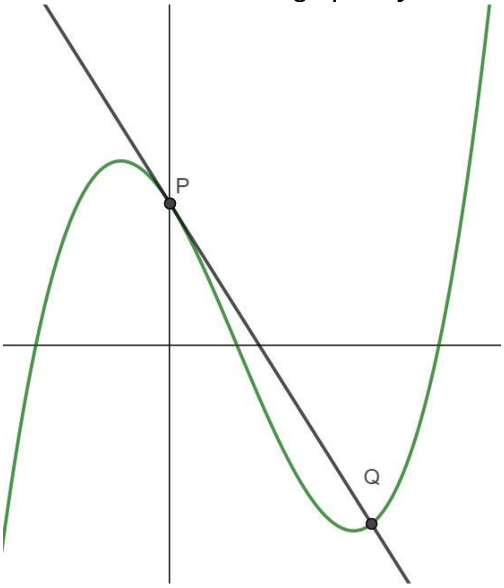
Let $f(x) = (x - 1)(x - 4)(x + 2)$. The diagram below shows the graph of $\boldsymbol{f}$ and the point P where the graph crosses the x axis.
The line L is the tangent to the graph of $\boldsymbol{f}$ at the point P.
The line L intersects the graph of $\boldsymbol{f}$ at another point Q, as shown below



a) Find the coordinates of P

b) Show that $f(x) = x^3 - 3x^2 - 6x + 8$

c) Find the equation of L in the form y = ax + b

d) Find the x coordinate of Q.

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a) Find the coordinates of P
$f(x) = (x - 1)(x - 4)(x + 2)$

Point P when x = 0. Find y coordinate by calculating f(0)

$f(0) = (-1)(-4)(2) = 8$

P(0 , 8)

b) Show that $f(x) = x^3 - 3x^2 - 6x + 8$

$f(x) = (x - 1)(x - 4)(x + 2)$

Expand the brackets. Start by expanding 1 pair of brackets.

$f(x) = (x - 1)(x^2 - 2x - 8)$

$f(x) = x(x^2 - 2x - 8) - 1(x^2 - 2x - 8)$

$f(x) = x^3 - 2x^2 - 8x - 1x^2 + 2x + 8$

$f(x) = x^3 - 3x^2 - 6x + 8$

c) Find the equation of L in the form y = ax + b

Find $f'(x)$

$f'(x) = 3x^2 - 6x - 6$

Find gradient a P

$f'(0) = 3 \cdot 0^2 - 6 \cdot 0 - 6$

$f'(0) = -6$

The tangent has gradient = -6

$y = -6x + c$

The tangent passes through the point (0 , 8)

$y = -6x + 8$

d) Find the x coordinate of Q.

Find the intersection of the tangent and the curve

$-6x + 8 = x^3 - 3x^2 - 6x + 8$

Write in the form g(x) = 0

$x^3 - 3x^2 = 0$

Factorise

$x^2(x - 3) = 0$

Solve

$x = 0\,,\ x = 3$

Hence the x coordinate of Q $= 3$