Given that $a x^{3}+b x^{2}+17 x-6$ is exactly divisible by $(x-1)(x-2)$, find the value of $a$ and the value of $b$.

$$
\text { Let } f(x)=a x^{3}+b x^{2}+17 x-6
$$

$(x-1)$ is a factor

$$
f(1)=0
$$

$$
\begin{aligned}
& f(1)=a(1)^{3}+b(1)^{2}+17(1)-6=0 \\
& a+b+17-6=0 \\
& a+b=-11
\end{aligned}
$$

$(x-2)$ is a factor $f(2)=0$

$$
\begin{aligned}
& f(2)=a(2)^{3}+b(2)^{2}+17(2)-6=0 \\
& 8 a+4 b+34-6=0 \\
& 8 a+4 b=-28 \\
& 2 a+b=-7
\end{aligned}
$$

$$
\begin{aligned}
& 2 a+b=-7 \\
& a+b=-11 \\
& a=4 \\
& b=-15
\end{aligned}
$$

