It is given that $f(x) = x^3 + ax^2 + bx + 8$

a) Given that $(x^2 - 4)$ is a factor of $f(x)$, find the value of a and the value of b.
b) Factorize $f(x)$ into a product of linear factors.
c) Sketch the graph of $y = f(x)$ labelling any stationary points and the x and y intercepts.
d) **Hence** state the range of values of ***k*** for which $f(x) = k$ has exactly one root.

a)

Let $f(x) = x^3 + ax^2 + bx + 8$

$(x^2 - 4)$ is a factor
Therefore $(x - 2)(x + 2)$ is a factor

$(x - 2)$ is a factor
$f(2) = 0$

$f(2) = (2)^3 + a(2)^2 + b(2) + 8 = 0$

$8 + 4a + 2b + 8 = 0$

$4a + 2b = -16$

$2a + b = -8$

$(x + 2)$ is a factor
$f(-2) = 0$

$f(-2) = (-2)^3 + a(-2)^2 + b(-2) + 8 = 0$

$-8 + 4a - 2b + 8 = 0$

$4a - 2b = 0$

$2a - b = 0$

$2a + b = -8$

$2a - b = 0$

$4a = -8$

$a = -2$

$b = -4$

b)

$$f(x) = x^3 - 2x^2 - 4x + 8$$
$$x^3 - 2x^2 - 4x + 8 = (x^2 - 4)(x - 2)$$
$$x^3 - 2x^2 - 4x + 8 = (x + 2)(x - 2)(x - 2)$$
$$x^3 - 2x^2 - 4x + 8 = (x + 2)(x - 2)^2$$

c)

x intercepts

$f(x) = 0$

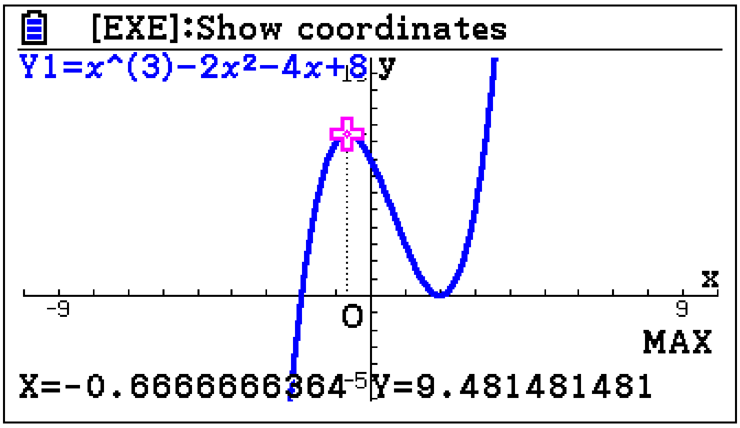
$$(x + 2)(x - 2)^2 = 0$$

$x = -2$, $x = 2$ (repeated root)

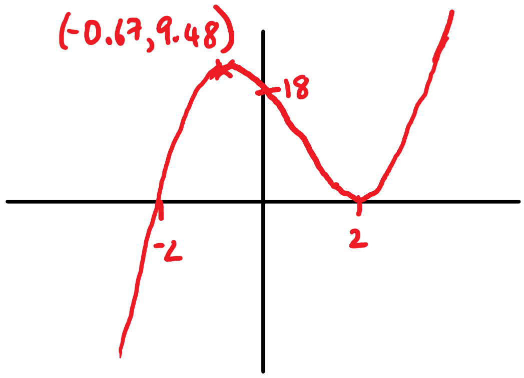
y intercepts

$$f(0) = 8$$

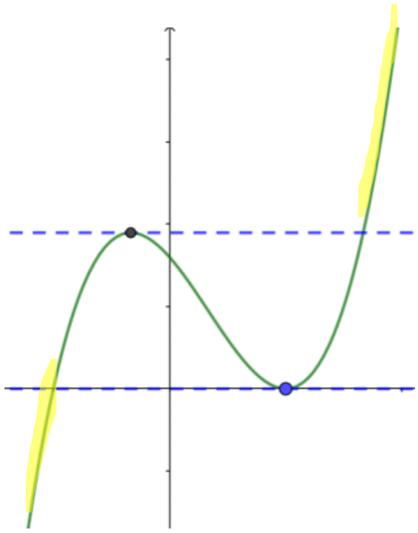
Use GDC to find the stationary points



Maximum at $(-0.67, 9.48)$ and $(2,0)$



d)



$f(x) = k$ has only one root

$k < 0$, $k > 9.48$