A quadratic function $\boldsymbol{f}$ can be written in the form $f(x) = a(x-h)^2 + k$. The graph of $\boldsymbol{f}$ has vertex $(-1,4)$ and has y-intercept at $(0,5)$.

a) Find the value of $\boldsymbol{a}$, $\boldsymbol{h}$ and $\boldsymbol{k}$.

b) The line $y = \boldsymbol{m}x + 1$ is a tangent to the curve $\boldsymbol{f}$. Find the value of $\boldsymbol{m}$.

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a) Since $f(x) = a(x-h)^2 + k$ gives us the vertex (h,k)

and the vertex is $(-1,4)$

then $f(x) = a(x+1)^2 + 4$

$h = -1 \, , k = 4$

The graph passes through the point $P(0,5)$

$x = 0 \, , y = 5$

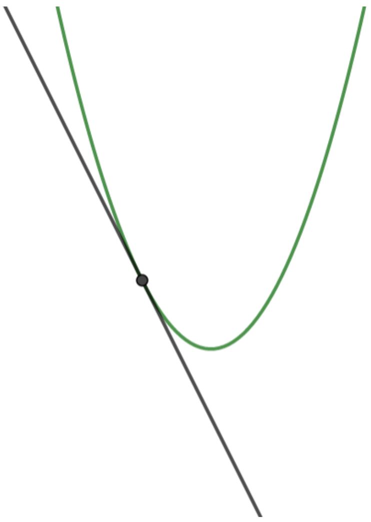
$5 = a(0+1)^2 + 4$

$5 = a + 4$

$a = 1$

$f(x) = (x+1)^2 + 4$

b)



If the line $y = mx + 1$ is a tangent to the curve $\boldsymbol{f}$

then the solution to $mx + 1 = (x+1)^2 + 4$

...has one repeated root

$$mx + 1 = x^2 + 2x + 1 + 4$$

$$0 = x^2 + 2x - mx + 4$$

$$0 = x^2 + (2-m)x + 4$$

If this quadratic has one repeated root

...then the discriminant, $\Delta = 0$

$$\Delta = (2-m)^2 - 4 \cdot 1 \cdot 4$$

$$\Delta = (2-m)^2 - 16$$

$$\Delta = 4 - 4m + m^2 - 16$$

$$\Delta = m^2 - 4m - 12$$

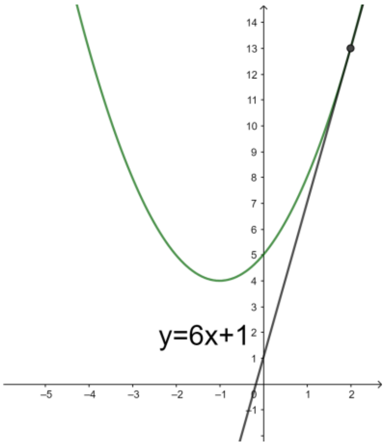
$$m^2 - 4m - 12 = 0$$

$$(m-6)(m+2) = 0$$

$$m = 6\,, m = -2$$

Here's what the solutions look like graphically

$m = 6$



$m = -2$