

$$\text{Let } f(x) = \frac{3x-2}{x-a}, x \neq a$$

a) Find the inverse function $f^{-1}(x)$ in terms of a

b) Find the value of a such that f is a self-inverse

a)

$$\text{Let } y = \frac{3x-2}{x-a}$$

Interchange x and y

$$x = \frac{3y-2}{y-a}$$

Make y the subject

$$x(y-a) = 3y-2$$

$$xy - ax = 3y - 2$$

$$xy - 3y = ax - 2$$

$$y(x-3) = ax - 2$$

$$y = \frac{ax-2}{x-3}$$

$$f^{-1}(x) = \frac{ax-2}{x-3}$$

b) If f is a self-inverse, then $f^{-1}(x) = f(x)$

$$\frac{ax-2}{x-3} = \frac{3x-2}{x-a}$$

$$a = 3$$