The line $l_{1}$ passes through the point $\mathrm{P}(3 \mathrm{k}, 2 \mathrm{k})$ with gradient $=-2$.
$l_{1}$ meets the x axis at A and the y axis at B .
a) Find the equation of the line $l_{1}$ and show that $A(4 k, 0)$
b) Find the area of the triangle AOB in terms of $k$

The line $l_{2}$ passes through P and is perpendicular to $l_{1}$.
c) Find the equation of $l_{2}$
$l_{2}$ meets the x axis at C
d) Show that the midpoint of PC lies on the line $y=x$
a) Drawing a sketch can be helpful to visualise the problem


Find the equation of the line $l_{1}$ in terms of $k$
$l_{1}$ passes through $(3 k, 2 k)$
with gradient $=-2$.

$$
y-2 k=-2(x-3 k)
$$

At point $A, y=0$

$$
0-2 k=-2(x-3 k)
$$

Divide both sides by -2

$$
\begin{aligned}
& k=x-3 k \\
& x=4 k \\
& A(4 k, 0)
\end{aligned}
$$

At point $B, x=0$

$$
\begin{aligned}
& y-2 k=-2(0-3 k) \\
& y-2 k=6 k \\
& y=8 k \\
& B(0,8 k)
\end{aligned}
$$

You can also work this out from the fact that the gradient of the line is -2
b)


Area $=\frac{1}{2} \times 4 k \times 8 k=16 k^{2}$

| c) |  |
| :---: | :---: |
|  | $\begin{aligned} & l_{2} \\ & \text { gradicat }=\frac{1}{2} \\ & (3 k, 2 k) \end{aligned}$ <br> A <br> gradient $=-2$ |
| $\begin{aligned} & l_{2} \text { passes through }(3 k, 2 k) \\ & \text { with gradient }=\frac{1}{2} \end{aligned}$ |  |
|  | $y-2 k=\frac{1}{2}(x-3 k)$ |
|  | $2 y-4 k=x-3 k$ |
|  | $y=\frac{1}{2} x+\frac{k}{2}$ |
| d) Let M be the midpoint of PC |  |
|  |  |
| At point $\mathrm{C}, \mathrm{y}=0$ | $y=\frac{1}{2} x+\frac{k}{2}$ |


|  | $0=\frac{1}{2} x+\frac{k}{2}$ |
| :--- | :--- |
|  | $0=x+k$ |
|  | $x=-k$ |
| Find midpoint of PC | $C(-k, 0)$ |
|  | $M\left(\frac{-k+3 k}{2}, \frac{0+2 k}{2}\right)$ |
|  | $M(k, k)$ |
| Since, x and y coordinates are equal, then M lies <br> on the line $y=x$ |  |

