

Let  $f$  and  $g$  be the functions

$$f(x) = \ln(1 - 2x), x < \frac{1}{2}$$

$$g(x) = \frac{4}{x - 2}, x \neq 2$$

a) Find the exact value of  $fg(-2)$

b) Find  $f^{-1}(x)$ , stating its domain

c) Show that  $gg(x) = \frac{2x-6}{11-3x}$

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a)  $f(x) = \ln(1 - 2x), x < \frac{1}{2}$

$$g(x) = \frac{4}{x - 2}, x \neq 2$$

$$g(-2) = \frac{4}{-2 - 2} = -1$$

$$fg(-2) = f(-1)$$

$$f(-1) = \ln(1 - 2(-1)) = \ln 3$$

b)  $y = \ln(1 - 2x)$

Interchange  $x$  and  $y$

$$x = \ln(1 - 2y)$$

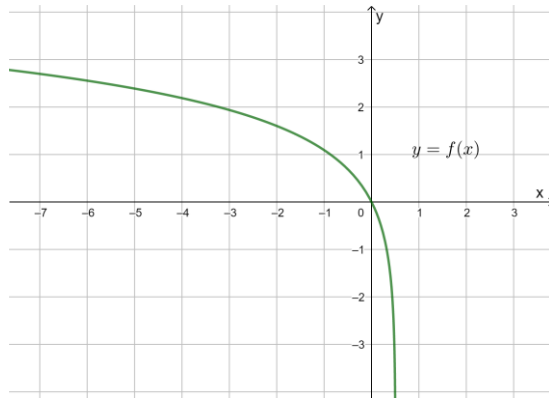
Make  $y$  the subject

$$e^x = 1 - 2y$$

$$2y = 1 - e^x$$

$$y = \frac{1 - e^x}{2}$$

Domain of  $f^{-1}(x)$  is range of  $f$



Range of  $f$  is  $f(x) \in \mathbb{R}$

$$f^{-1}(x) = \frac{1 - e^x}{2}, x \in \mathbb{R}$$

c) 
$$g(x) = \frac{4}{x-2}$$

$$g(g(x)) = g\left(\frac{4}{x-2}\right)$$

$$g(g(x)) = \frac{4}{\frac{4}{x-2} - 2}$$

$$g(g(x)) = \frac{4}{\frac{4 - 2(x-2)}{x-2}}$$

$$g(g(x)) = \frac{4}{\frac{4 - 2x + 4}{x-2}}$$

$$g(g(x)) = \frac{4}{\frac{8 - 2x}{x-2}}$$

$$g(g(x)) = \frac{4(x-2)}{8-2x}$$

$$g(g(x)) = \frac{4x-8}{8-2x}$$