One root of the equation $4z^4 - 4z^3 - 25z^2 + 55z - 42 = 0$ is $1 + \frac{\sqrt{3}}{2}i$. Find the other roots of the equation.

If
$$z = 1 + \frac{\sqrt{3}}{2}i$$
 is a root ...then, the complex conjugate $z^* = 1 - \frac{\sqrt{3}}{2}i$ is also a root

Factors of the equation are $z - \left(1 + \frac{\sqrt{3}}{2}i\right)$

$$z = \left(1 + \frac{\sqrt{3}}{2}i\right)$$
$$z = \left(1 - \frac{\sqrt{3}}{2}i\right)$$

The polynomial equation is

is
$$a(x-\alpha)(x-\beta)\left(z-\left(1+\frac{\sqrt{3}}{2}i\right)\right)\left(z-\left(1-\frac{\sqrt{3}}{2}i\right)\right)=0$$

 $\left(z-\left(1+\frac{\sqrt{3}}{2}i\right)\right)\left(z-\left(1-\frac{\sqrt{3}}{2}i\right)\right)$
 $=z^{2}-\left(1-\frac{\sqrt{3}}{2}i\right)z-\left(1+\frac{\sqrt{3}}{2}i\right)z+\left(1+\frac{\sqrt{3}}{2}i\right)\left(1-\frac{\sqrt{3}}{2}i\right)$
 $=z^{2}-2z+1-\frac{3}{4}i^{2}$
 $=z^{2}-2z+\frac{7}{4}$

$$\begin{aligned} 4z^4 - 4z^3 - 25z^2 + 55z - 42 &= a(z - \alpha)(z - \beta)\left(z^2 - 2z + \frac{7}{4}\right) \\ &= (z - \alpha)(z - \beta)(4z^2 - 8z + 7) \\ &= (z^2 + bz - 6)(4z^2 - 8z + 7) \end{aligned}$$

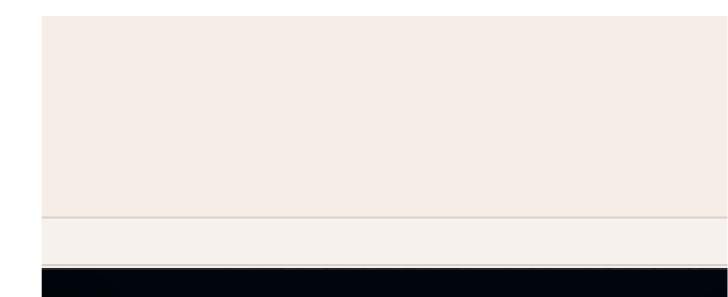
$$-4z^3 = -8z^3 + 4bz^3 \ b = 1$$

$$4z^4 - 4z^3 - 25z^2 + 55z - 42 = (z^2 + z - 6)(4z^2 - 8z + 7)$$

 $4z^{4} - 4z^{3} - 25z^{2} + 55z - 42 = 0$ $4z^{2} - 8z + 7 = 0$ $z = 1 + \frac{\sqrt{3}}{2}i, z = 1 - \frac{\sqrt{3}}{2}i$ $z^{2} + z - 6 = 0$ (z + 3)(z - 2) = 0 z = -3, z = 2

Other roots
$$z = 1 - \frac{\sqrt{3}}{2}i, z = -3, z = 2$$





Complex Roots of Polynomials Page 3