

In the quadratic equation $px^2 - 45x + 25 = 0$, $p \in \mathbb{Z}$, one root is two times the other.

Find the value of p .

$$px^2 - 45x + 25 = 0$$

$$\begin{aligned} \text{Sum of roots} &= \frac{45}{p} \\ \text{Product of roots} &= \frac{25}{p} \end{aligned}$$

Let the roots be $\alpha, 2\alpha$

$$\begin{aligned} \text{Sum of roots} &= \alpha + 2\alpha = 3\alpha \\ \text{Product of roots} &= \alpha \times 2\alpha = 2\alpha^2 \end{aligned}$$

$$\begin{aligned} \frac{45}{p} = 3\alpha &\Rightarrow p = \frac{15}{\alpha} \\ \frac{25}{p} = 2\alpha^2 &\Rightarrow p = \frac{25}{2\alpha^2} \end{aligned}$$

$$\frac{15}{\alpha} = \frac{25}{2\alpha^2}$$

$$\frac{\alpha^2}{\alpha} = \frac{25}{30}$$

$$\alpha = \frac{5}{6}$$

$$p = \frac{15}{\frac{5}{6}}$$

$$p = 18$$