Exam-style Question

$$\int e^{\frac{x}{2}} \cdot \sin x \, dx =$$

$$\int u \cdot \frac{dv}{dx} \, dx = uv - \int v \cdot \frac{du}{dx} \, dx$$

Method 1

$u = \sin x$ $\quad \frac{dv}{dx} = e^{\frac{x}{2}}$

$\frac{du}{dx} = \cos x$ $\quad v = 2e^{\frac{x}{2}}$

- Logs
- Inverse Trig
- Algebra
- Trig / Exponential

$$\int e^{\frac{x}{2}} \cdot \sin x \, dx = (\sin x)(2e^{\frac{x}{2}}) - \int (2e^{\frac{x}{2}})(\cos x) \, dx$$

$$= 2e^{\frac{x}{2}} \sin x - \left[(\cos x)(4e^{\frac{x}{2}}) - \int (4e^{\frac{x}{2}})(-\sin x) \, dx \right]$$

$u = \cos x$ $\quad \frac{dv}{dx} = 2e^{\frac{x}{2}}$

$\frac{du}{dx} = -\sin x$ $\quad v = 4e^{\frac{x}{2}}$

$$= 2e^{\frac{x}{2}} \sin x - 4e^{\frac{x}{2}} \cos x - \int 4e^{\frac{x}{2}} \sin x \, dx$$

$$I = 2e^{\frac{x}{2}} \sin x - 4e^{\frac{x}{2}} \cos x - 4I$$

$$5I = 2e^{\frac{x}{2}} \sin x - 4e^{\frac{x}{2}} \cos x$$

$$I = \frac{2e^{\frac{x}{2}} \sin x - 4e^{\frac{x}{2}} \cos x}{5} + C$$

Method 2

$$\int e^{\frac{x}{2}} \cdot \sin x \, dx =$$

$u = e^{\frac{x}{2}}$ $\frac{dv}{dx} = \sin x$

$\frac{du}{dx} = \frac{1}{2} e^{\frac{x}{2}}$ $v = -\cos x$

$$\int u \cdot \frac{dv}{dx} \, dx = uv - \int v \cdot \frac{du}{dx} \, dx$$

$$= \left(e^{\frac{x}{2}}\right)(-\cos x) - \int (-\cos x)\left(\tfrac{1}{2} e^{\frac{x}{2}}\right) dx$$

$$= -e^{\frac{x}{2}} \cos x + \int \tfrac{1}{2} e^{\frac{x}{2}} \cos x \, dx$$

$u = \frac{1}{2} e^{\frac{x}{2}}$ $\frac{dv}{dx} = \cos x$

$\frac{du}{dx} = \frac{1}{4} e^{\frac{x}{2}}$ $v = \sin x$

$$= -e^{\frac{x}{2}} \cos x + \left(\tfrac{1}{2} e^{\frac{x}{2}}\right)(\sin x) - \int (\sin x)\left(\tfrac{1}{4} e^{\frac{x}{2}}\right) dx$$

$$= -e^{\frac{x}{2}} \cos x + \tfrac{1}{2} e^{\frac{x}{2}} \sin x - \int \tfrac{1}{4} e^{\frac{x}{2}} \sin x \, dx$$

$$I = -e^{\frac{x}{2}} \cos x + \tfrac{1}{2} e^{\frac{x}{2}} \sin x - \tfrac{1}{4} I$$

$$\tfrac{5}{4} I = -e^{\frac{x}{2}} \cos x + \tfrac{1}{2} e^{\frac{x}{2}} \sin x$$

$$I = \frac{-4e^{\frac{x}{2}} \cos x + 2e^{\frac{x}{2}} \sin x}{5} + C$$