Integration by Parts

Integration by parts is a method of integration that we use to integrate the product (usually !) of two functions. The aim is to change this product into another one that is easier to integrate.

Here is the formula from the booklet $\int u \cdot \frac{dv}{dx}\, dx \;=\; uv \;-\; \int v \cdot \frac{du}{dx}\, dx$

- Whenever you get a question that you think requires integration by parts first check that you cannot do it by integration by substitution (or recognition). See the page on this topic for help with this.
- You then need to decide which part of the question we call ***u*** and which part $\frac{dv}{dx}$. Look at the two functions in your question. The one that appears first on the following list is the one you should choose to be ***u***

    1. Logs
    2. Inverse Trig
    3. Algebra
    4. Trig / Exponential

- It is really useful to have your 'formula booklet' open so that you can refer to rules for integration and differentiation.

- Sometimes you have to apply integration by parts twice for questions like this
  $\int x^2 \cdot \sin\left(\frac{x}{2}\right) dx$ or $\int x^2 \cdot e^x\, dx$
  Extra care is required for these (check your working and look out for changes of sign).

- You can use integration by parts to integrate ln(x) by changing it into a product of two functions: $\int 1 \cdot lnx\, dx$
  This method is also used for arcsin(x) , arccos(x) and arctan(x). It is a good idea to learn these by heart.

- Sometimes questions require you to use integration by parts and also integration by substitution or recognition. It is important to be comfortable with all these methods. Her's an example

$$\int 1 \cdot arctanx\, dx$$

$$u = arctanx \qquad \frac{dv}{dx} = 1$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \qquad v = x$$

$$\int 1 \cdot arctanx\, dx = x \cdot arctanx - \int \frac{x}{1+x^2} dx$$

Use integration by recognition: $\int \frac{f\prime(x)}{f(x)}\, dx \;=\; ln|f(x)| \;+ C$

$$\int 1 \cdot arctanx\, dx = \; x \cdot arctanx - \frac{1}{2}\int \frac{2x}{1+x^2} dx$$

$$= x \cdot arctanx - \frac{1}{2} ln|1+x^2| + C$$