a) Using the fact that $tanx = \frac{sinx}{cosx}$, show that $\frac{d}{dx}(tanx) = \frac{1}{cos^2 x}$

b) **Hence**, find $\int \frac{\sqrt{tanx}}{cos^2 x} dx$

a) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$ ← Quotient

Quotient Rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = \sin x$ $\quad$ $v = \cos x$

$\frac{du}{dx} = \cos x$ $\quad$ $\frac{dv}{dx} = -\sin x$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$$

b) $\int \frac{\sqrt{\tan x}}{\cos^2 x} dx$

Integration by Substitution

$u = \tan x$

$\frac{du}{dx} = \frac{1}{\cos^2 x}$

$du = \frac{1}{\cos^2 x} dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}} du$

$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3}(\tan x)^{3/2} + C$

$= \frac{2}{3}\sqrt{\tan^3 x} + C$