Find $\int \frac{arcsin x + x}{\sqrt{1-x^2}} dx$

Split into 2 integrals

$$= \int \frac{\arcsin x}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$$

Integration by substitution

$$u = \arcsin x$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$du = \frac{1}{\sqrt{1-x^2}} dx$$

Integration by substitution

$$v = x^2$$

$$\frac{dv}{dx} = 2x$$

$$\frac{1}{2} dv = x dx$$

$$= \int u \, du + \int \frac{\frac{1}{2} dv}{\sqrt{1-v}}$$

$$= \int u \, du + \frac{1}{2} \int (1-v)^{-\frac{1}{2}} dv$$

Remember standard integral:

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$= \frac{u^2}{2} + \frac{1}{2} \cdot \frac{(1-v)^{\frac{1}{2}}}{\frac{1}{2}(-1)} + C$$

$$= \frac{u^2}{2} - \sqrt{1-v} + C$$

$$= \frac{\arcsin^2 x}{2} - \sqrt{1-x^2} + C$$