Find $\int \sin^5 x \, dx$

$$= \int \sin^4 x \cdot \sin x \, dx$$

$$= \int (\sin^2 x)^2 \sin x \, dx$$

$$= \int (1 - \cos^2 x)^2 \sin x \, dx$$

$$= \int (1 - u^2)^2 \cdot -1 \, du$$

$$= \int -(1-u^2)(1-u^2) \, du$$

$$= \int -(1 - u^2 - u^2 + u^4) \, du$$

$$= \int -(1 - 2u^2 + u^4) \, du$$

$$= \int (-1 + 2u^2 - u^4) \, du$$

$$= -u + \frac{2u^3}{3} - \frac{u^5}{5} + C$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{\cos^5 x}{5} + C$$

$\sin^2 x + \cos^2 x \equiv 1$

$\sin^2 x \equiv 1 - \cos^2 x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-1 \cdot du = \sin x \, dx$