$\int \sqrt{16 - 9x^2}\, dx$ using the substitution $3x = 4\sin\theta$

$x = \frac{4}{3}\sin\theta$

$\frac{dx}{d\theta} = \frac{4}{3}\cos\theta$

$dx = \frac{4}{3}\cos\theta \, d\theta$

$(3x)^2 = 9x^2$

$= (4\sin\theta)^2$

$= 16\sin^2\theta$

$$= \int \sqrt{16 - 16\sin^2\theta}\ \frac{4}{3}\cos\theta \, d\theta$$

$$= \int \sqrt{16(1 - \sin^2\theta)} \cdot \frac{4}{3}\cos\theta \, d\theta$$

$\sin^2\theta + \cos^2\theta \equiv 1$

$\cos^2\theta \equiv 1 - \sin^2\theta$

$$= \int \sqrt{16\cos^2\theta} \cdot \frac{4}{3}\cos\theta \, d\theta$$

$$= \int 4\cos\theta \cdot \frac{4}{3}\cos\theta \, d\theta$$

$$= \int \frac{16}{3}\cos^2\theta \, d\theta$$

$\cos 2\theta \equiv 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$$= \int \frac{8}{3}\, 2\cos^2\theta \, d\theta$$

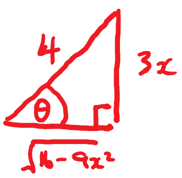
$$= \int \frac{8}{3}(\cos 2\theta + 1)\, d\theta$$

$$= \frac{8}{3}\int (\cos 2\theta + 1)\, d\theta$$

$$= \frac{8}{3}\left(\frac{\sin 2\theta}{2} + \theta\right) + C$$

$3x = 4\sin\theta$

$\frac{3x}{4} = \sin\theta$



$\sin 2\theta \equiv 2\sin\theta\cos\theta$

$= 2 \cdot \frac{3x}{4} \cdot \frac{\sqrt{16-9x^2}}{4}$

$= \frac{6x}{16}\sqrt{16-9x^2}$

$= \frac{3x}{8}\sqrt{16-9x^2}$

$$= \frac{8}{3}\left(\frac{\frac{3x}{8}\sqrt{16-9x^2}}{2} + \arcsin\left(\frac{3x}{4}\right)\right) + C$$

$$= \frac{x}{2}\sqrt{16-9x^2} + \frac{8}{3}\arcsin\left(\frac{3x}{4}\right) + C$$