

$$\int \sqrt{16 - 9x^2} dx \text{ using the substitution } 3x = 4 \sin \theta$$

$$= \int \sqrt{16 - 16 \sin^2 \theta} \cdot \frac{4 \cos \theta}{3} d\theta$$

$$= \int \sqrt{16(1 - \sin^2 \theta)} \cdot \frac{4}{3} \cos \theta d\theta$$

$$= \int \sqrt{16 \cos^2 \theta} \cdot \frac{4}{3} \cos \theta d\theta$$

$$= \int 4 \cos \theta \cdot \frac{4}{3} \cos \theta d\theta$$

$$= \int \frac{16}{3} \cos^2 \theta d\theta$$

$$= \int \frac{8}{3} 2 \cos^2 \theta d\theta$$

$$= \int \frac{8}{3} (\cos 2\theta + 1) d\theta$$

$$= \frac{8}{3} \int (\cos 2\theta + 1) d\theta$$

$$= \frac{8}{3} \left( \frac{\sin 2\theta}{2} + \theta \right) + C$$

$$= \frac{8}{3} \left( \frac{3x}{8} \frac{\sqrt{16-9x^2}}{2} + \arcsin\left(\frac{3x}{4}\right) \right) + C$$

$$= \frac{x}{2} \sqrt{16-9x^2} + \frac{8}{3} \arcsin\left(\frac{3x}{4}\right) + C$$

$$x = \frac{4}{3} \sin \theta$$

$$\frac{dx}{d\theta} = \frac{4}{3} \cos \theta$$

$$dx = \frac{4}{3} \cos \theta d\theta$$

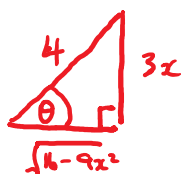
$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &\equiv 1 \\ \cos^2 \theta &\equiv 1 - \sin^2 \theta \end{aligned}$$

$$\begin{aligned} (3x)^2 &= 9x^2 \\ &= (4 \sin \theta)^2 \\ &= 16 \sin^2 \theta \end{aligned}$$

$$\begin{aligned} \cos 2\theta &\equiv 2 \cos^2 \theta - 1 \\ \cos 2\theta + 1 &= 2 \cos^2 \theta \end{aligned}$$

$$3x = 4 \sin \theta$$

$$\frac{3x}{4} = \sin \theta$$



$$\begin{aligned} \sin 2\theta &\equiv 2 \sin \theta \cos \theta \\ &= 2 \cdot \frac{3x}{4} \cdot \frac{\sqrt{16-9x^2}}{4} \\ &= \frac{6x}{16} \sqrt{16-9x^2} \\ &= \frac{3x}{8} \sqrt{16-9x^2} \end{aligned}$$