Given that $\int_2^5 \ln(sinx)\, dx = A$

show that $\int_2^5 \ln(e^x sinx)\, dx = A + \frac{21}{2}$

The following log law is useful:

$$\log_a xy = \log_a x + \log_a y$$

$$\int_2^5 \ln(e^x \cdot sinx)\, dx = \int_2^5 [\ln(e^x) + \ln(sinx)]dx$$

Use this property of the definite integral

$$\int_a^b (f(x) \pm g(x))dx$$

$$= \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_2^5 [\ln(e^x) + \ln(sinx)]dx$$

$$= \int_2^5 \ln(e^x)\, dx + \int_2^5 \ln(sinx)dx$$

$$\ln(e^x) = x$$

$$= \int_2^5 xdx + \int_2^5 \ln(sinx)dx$$

$$= \left[\frac{x^2}{2}\right]_2^5 + A$$

$$= \left[\frac{5^2}{2}\right] - \left[\frac{2^2}{2}\right] + A$$

$$= \frac{25}{2} - \frac{4}{2} + A$$

$$= \frac{21}{2} + A$$