$$3xy^2\frac{dy}{dx} = x^3 + y^3$$

$$\frac{dy}{dx} = \frac{x^3 + y^3}{3xy^2}$$

$$\frac{dy}{dx} = \frac{\frac{x^3}{x^3} + \frac{y^3}{x^3}}{\frac{3xy^2}{x^3}}$$

$$\frac{dy}{dx} = \frac{1 + \left(\frac{y}{x}\right)^3}{3\left(\frac{y}{x}\right)^2}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1 + (v)^3}{3(v)^2}$$

$$x\frac{dv}{dx} = \frac{1 + v^3}{3v^2} - v$$

$$x\frac{dv}{dx} = \frac{1 + v^3}{3v^2} - \frac{3v^3}{3v^2}$$

$$x\frac{dv}{dx} = \frac{1 - 2v^3}{3v^2}$$

$$\int \frac{3v^2}{1 - 2v^3}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\int \frac{-6v^2}{1 - 2v^3}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\ln|1 - 2v^3| = \ln|x| + C$$

$$\ln(1 - 2v^3)^{-\frac{1}{2}} = \ln|x| + lnk$$

$$\ln(1 - 2v^3)^{-\frac{1}{2}} = \ln|kx|$$

$$(1 - 2v^3)^{-\frac{1}{2}} = kx$$

$$\frac{1}{\sqrt{1-2v^3}} = kx$$

$$\sqrt{1-2v^3} = \frac{1}{kx}$$

$$1-2v^3 = \frac{1}{k^2 x^2}$$

$$2v^3 = 1 - \frac{1}{k^2 x^2}$$

$$v^3 = \frac{1}{2} - \frac{1}{2k^2 x^2}$$

$$v^3 = \frac{k^2 x^2 - 1}{2k^2 x^2}$$

$$v = \sqrt[3]{\frac{k^2 x^2 - 1}{2k^2 x^2}}$$

$$\frac{y}{x} = \sqrt[3]{\frac{k^2 x^2 - 1}{2k^2 x^2}}$$

$$y = x\sqrt[3]{\frac{k^2 x^2 - 1}{2k^2 x^2}}$$

$$y = \sqrt[3]{\frac{x^3(k^2 x^2 - 1)}{2k^2 x^2}}$$

$$y = \sqrt[3]{\frac{x(k^2 x^2 - 1)}{2k^2}}$$