$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\frac{dy}{dx} + \frac{y}{x} = sinx$$

$$\frac{dy}{dx} + \frac{1}{x}y = sinx$$

integrating factor $I = e^{\int P(x)dx}$

$$I = e^{\int \frac{1}{x} dx}$$

$$I = e^{lnx}$$

$$I = x$$

We multiply the differential equation through by $x$

$$x\frac{dy}{dx} + x\frac{1}{x}y = xsinx$$

$$x\frac{dy}{dx} + 1 \cdot y = xsinx$$

Product Rule

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(x \cdot y) = x\frac{dy}{dx} + 1 \cdot y$$

$$\frac{d}{dx}(x \cdot y) = xsinx$$

$$\frac{d}{dx}(x \cdot y) = x\sin x$$

$$\int \frac{d}{dx}(x \cdot y)\, dx = \int x\sin x\, dx$$

$$x \cdot y = \int x\sin x\, dx$$

*Integration by Parts*

$$\int u \cdot \frac{dv}{dx}\, dx = uv - \int v \cdot \frac{du}{dx}\, dx$$

$$u = x \qquad\qquad \frac{dv}{dx} = \sin x$$
$$\frac{du}{dx} = 1 \qquad\qquad v = -\cos x$$

$$x \cdot y = x(-\cos x) - \int (-\cos x) \cdot 1 \, dx$$

$$x \cdot y = -x\cos x + \int \cos x \, dx$$

$$x \cdot y = -x\cos x + \sin x + C$$

$$y = -\cos x + \frac{\sin x}{x} + \frac{C}{x}$$