$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$cosx\frac{dy}{dx} + ysinx = sin2x$$

$$\frac{dy}{dx} + \frac{sinx}{cosx}y = \frac{sin2x}{cosx}$$

integrating factor $I = e^{\int P(x)dx}$

$$I = e^{\int \frac{sinx}{cosx}dx}$$

$$I = e^{-\int \frac{-sinx}{cosx}dx}$$

$$I = e^{-ln|cosx|}$$

$$I = e^{ln\left|\frac{1}{cosx}\right|}$$

$$I = \frac{1}{cosx}$$

We multiply the differential equation through by $\frac{1}{cosx}$

$$\frac{1}{cosx}\frac{dy}{dx} + \frac{1}{cosx}\frac{sinx}{cosx}y = \frac{1}{cosx}\frac{sin2x}{cosx}$$

$$\frac{1}{cosx}\frac{dy}{dx} + \frac{sinx}{cos^2x}\cdot y = \frac{2sinxcosx}{cos^2x}$$

$$\frac{1}{cosx}\frac{dy}{dx} + \frac{sinx}{cos^2x}\cdot y = \frac{2sinx}{cosx}$$

Product Rule

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{cosx}\cdot y\right) = \frac{sinx}{cos^2x}\cdot y + \frac{1}{cosx}\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{cosx}\cdot y\right) = \frac{2sinx}{cosx}$$

$$\int \frac{d}{dx}\left(\frac{1}{cosx}\cdot y\right)dx = \int \frac{2sinx}{cosx}dx$$

$$\frac{y}{cosx} = -2\int \frac{-sinx}{cosx}dx$$

$$y = -2cosx\ln|cosx| + Ccosx$$

INTHINKING

$y(0) = 2$

$2 = -2cos0\ln|cos0| + Ccos0$

$2 = -2 \cdot 1 \ln 1 + C \cdot 1$

$2 = 0 + C$

$C = 2$

$y = -2cosx \ln|cosx| + 2cosx$