Integrating Factor Differential Equations

There are several steps involved in this method…

A differential equation that can be solved using an integrating factor must be in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

The integrating factor, I, is a function that we get by working out

$$I = e^{\int P(x)dx}$$

We multiply the differential equation through by I:

$$e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y = e^{\int P(x)dx}Q(x)$$

The left-hand side of this equation is the derivative of the product of two functions:

$$e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y = \frac{d}{dx}\left(e^{\int P(x)dx}y\right)$$

So, the equation becomes:

$$\frac{d}{dx}\left(e^{\int P(x)dx}y\right) = e^{\int P(x)dx}Q(x)$$

And we can solve this equation by integrating both sides of this equation:

$$e^{\int P(x)dx}y = \int \left(e^{\int P(x)dx}Q(x)\right)dx$$

Once we have worked out this integral, we divide both sides through by $e^{\int P(x)dx}$